

HW4-1

2023年3月28日 11:51

$$40. f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{Other} \end{cases}$$

$$\begin{aligned} (1) P(Y_1 = y) &= P(e^X = y) \\ &= P(X = \ln y) \\ &= f(\ln y) \frac{1}{y} dy \end{aligned}$$

$$f_{Y_1}(y) = \begin{cases} \frac{1}{y}, & 1 < y < e \\ 0, & \text{Other} \end{cases}$$

$$\begin{aligned} (2) P(Y_2 = y) &= P(X^{-1} = y) \\ &= P(X = \frac{1}{y}) \\ &= f(\frac{1}{y}) \frac{1}{y^2} dy \end{aligned}$$

$$f_{Y_2}(y) = \begin{cases} \frac{1}{y^2}, & x > 1 \\ 0, & x \leq 1 \end{cases}$$

$$\begin{aligned} (3) P(Y_3 = y) &= P(-\frac{1}{\lambda} \ln X = y) \\ &= P(X = e^{-\lambda y}) \\ &= f(e^{-\lambda y}) \lambda e^{-\lambda y} dy \end{aligned}$$

$$\therefore f_{Y_3}(y) = \begin{cases} \lambda e^{-\lambda y}, & y > 0 \\ 0, & y \leq 0 \end{cases}$$

42. $Y \in (0, 1)$, 故 $\exists \gamma \in (0, 1)$ 使

$$\begin{aligned} P(Y \leq y) &= P(F(X) \leq y) = P(X \leq F^{-1}(y)) \\ &= F(F^{-1}(y)) \\ &= y \end{aligned}$$

$$\therefore Y \sim U(0, 1)$$

$$44. P(Y = y) \quad \text{令 } Y = g(X)$$

$$\begin{aligned} &= P(g(X) = y) \\ &= P(X = g^{-1}(y)) \end{aligned}$$

$$\text{令 } g^{-1}(y) = \varphi(y)$$

$$\begin{aligned} &= P(X = \varphi(y)) \\ &= f(\varphi(y)) \varphi'(y) dy \end{aligned}$$

$$\therefore f_Y(y) = 2(1 - \varphi(y)) \varphi'(y) = e^{-y}, \text{ 且 } \varphi(0) = 0$$

$$\text{解得, } x = \varphi(y) = 1 - e^{-\frac{1}{2}y}$$

$$\therefore y = g(x) = -2 \ln(1-x)$$

$$48. \int_{-\infty}^{+\infty} f(x) dx = \frac{9}{a} = 1$$

$$\therefore a = 9$$

$$\therefore f(x) = \frac{1}{9} x^2$$

1 < y < 2 时

$$F_Y(y) = P(Y=1) + P(1 < Y \leq y)$$

$$= P(X > 2) + P(1 < X \leq y)$$

$$= \int_2^3 f(x) dx + \int_1^y f(x) dx$$

$$= \frac{y^3 + 18}{27}$$

$$\therefore F_Y(y) = \begin{cases} 0, & y < 1 \\ \frac{y^3 + 18}{27}, & 1 \leq y \leq 2 \\ 1, & y > 2 \end{cases}$$

$$\begin{aligned} (2) P(X \leq Y) &= P(X \leq Y | X \leq 1) P(X \leq 1) \\ &\quad + P(X \leq Y | 1 < X < 2) P(1 < X < 2) \\ &\quad + P(X \leq Y | X \geq 2) P(X \geq 2) \\ &= 1 \times \int_0^1 f(x) dx + 1 \times \int_1^2 f(x) dx + 0 \\ &= \int_0^2 f(x) dx \\ &= \frac{8}{27} \end{aligned}$$

$$49. (1) y = \frac{x}{1-x} \quad x \in (0, 1), y \in (0, +\infty)$$

$$x = \frac{y}{1+y}$$

$$f_Y(y) = f_X\left(\frac{y}{1+y}\right) \left|\left(\frac{y}{1+y}\right)'\right|$$

$$= \frac{1}{(1+y)^2} \quad (y > 0)$$

$$(2) Z = \begin{cases} X, & a < X < 1 \\ 0, & 0 < X \leq a \end{cases}$$

$$P(Z \leq z) = \begin{cases} 0, & z < 0 \\ a, & 0 \leq z < a \\ z, & a < z < 1 \\ 1, & z \geq 1 \end{cases}$$

(3) 不会.